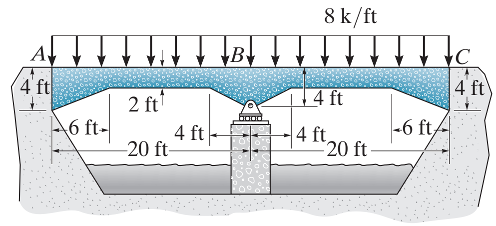
13–1. Determine the moments at A, B, and C by the moment-distribution method. Assume the supports at A and C are fixed and a roller support at B is on a rigid base. The girder has a thickness of 4 ft. Use Table 13–1. E is constant. The haunches are straight.



$$a_A = \frac{6}{20} = 0.3 \qquad a_B = \frac{4}{20} = 0.2$$

$$r_A = r_B = \frac{4-2}{2} = 1$$

From Table 13–1,

For span AB,

$C_{AB} = 0.622 \qquad C_{BA} = 0.748$

$K_{AB} = 10.06 \qquad K_{BA} = 8.37$

$$K_{BA} = \frac{K_{BA}EI_C}{L} = \frac{8.37EI_C}{20} = 0.4185EI_C$$

$(\text{FEM})_{AB} = -0.1089(8)(20)^2 = -348.48 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{BA} = 0.0942(8)(20)^2 = 301.44 \text{ k}\cdot\text{ft}$

For span BC,

$C_{BC} = 0.748 \qquad C_{CB} = 0.622$

$K_{BC} = 8.37 \qquad K_{CB} = 10.06$

$K_{BC} = 0.4185EI_C$

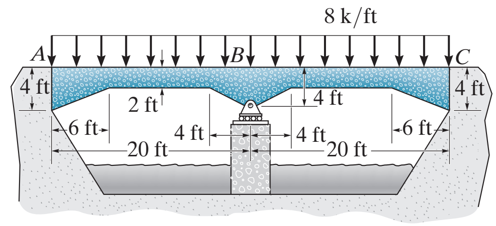
$(\text{FEM})_{BC} = -301.44 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{CB} = 348.48 \text{ k}\cdot\text{ft}$

Joint	A	B		C
Mem.	AB	BA	BC	CB
K		$0.4185EI_C$	$0.4185EI_C$	
DF	0	0.5	0.5	0
COF	0.622	0.748	0.748	0.622
FEM	–348.48	301.44	–301.44	348.48
		0	0	
$\sum M$	–348.48	301.44	–301.44	348.48 k·ft

Ans.

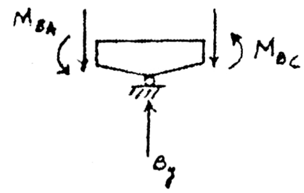
13–2. Solve Prob. 13–1 using the slope-deflection equations.



$$a_A = \frac{6}{20} = 0.3 \qquad a_B = \frac{4}{20} = 0.2$$

$$r_A = r_B = \frac{4-2}{2} = 1$$

For span AB,



$C_{AB} = 0.622 \qquad C_{BA} = 0.748$

$K_{AB} = 10.06 \qquad K_{BA} = 8.37$

$$K_{BA} = \frac{K_{BA}EI_C}{L} = \frac{8.37EI_C}{20} = 0.4185EI_C$$

$(\text{FEM})_{AB} = -0.1089(8)(20)^2 = -348.48\ \text{k}\cdot\text{ft}$

$(\text{FEM})_{BA} = 0.0942(8)(20)^2 = 301.44\ \text{k}\cdot\text{ft}$

For span BC,

$C_{BC} = 0.748 \qquad C_{CB} = 0.622$

$K_{BC} = 8.37 \qquad K_{CB} = 10.06$

$K_{BC} = 0.4185EI_C$

$(\text{FEM})_{BC} = -301.44\ \text{k}\cdot\text{ft}$

$(\text{FEM})_{CB} = 348.48\ \text{k}\cdot\text{ft}$

$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 + C_N)] + (\text{FEM})_N$

$M_{AB} = 0.503EI(0 + 0.622\theta_B - \ ) - 348.48$

$M_{AB} = 0.312866EI\theta_B - 348.8 \qquad (1)$

$M_{BA} = 0.4185EI(\theta_B + 0 - 0) + 301.44$

$M_{BA} = 0.4185EI\theta_B + 301.44 \qquad (2)$

$M_{BC} = 0.4185EI(\theta_B + 0 - 0) - 301.44$

$M_{BC} = 0.4185EI\theta_B - 301.44 \qquad (3)$

$M_{CB} = 0.503EI(0 + 0.622\theta_B - 0) + 348.48$

$M_{CB} = 0.312866EI\theta_B - 348.48 \qquad (4)$

Equilibrium.

$M_{BA} + M_{BC} = 0 \qquad (5)$

Solving Eqs. 1–5:

$\theta_B = 0$

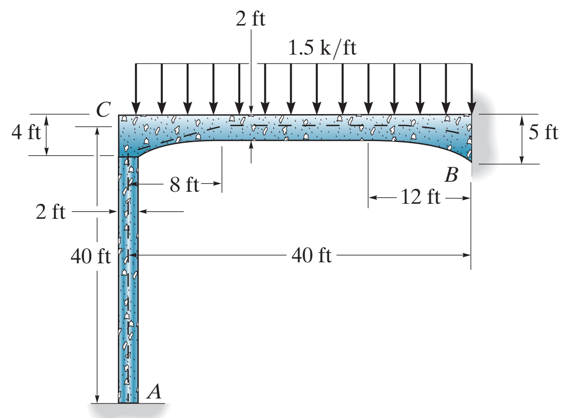
$M_{AB} = -348\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BA} = 301\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -301\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 348\ \text{k}\cdot\text{ft}$ **Ans.**

13–3. Apply the moment-distribution method to determine the moment at each joint of the parabolic haunched frame. Supports *A* and *B* are fixed. Use Table 13–2. The members are each 1 ft thick. *E* is constant.



The necessary data for member *BC* can be found from Table 13–2.

Here,

$$a_C = \frac{8}{40} = 0.2 \qquad a_B = \frac{12}{40} = 0.3 \qquad r_C = \frac{4-2}{2} = 1.0 \qquad r_B = \frac{5-2}{2} = 1.5$$

Thus,

$$C_{CB} = 0.735 \qquad C_{BC} = 0.589 \qquad K_{CB} = 7.02 \qquad K_{BC} = 8.76$$

Then,

$$K_{CB} = \frac{K_{CB}EI_C}{L_{BC}} = \frac{7.02E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.117E$$

The fixed end moment are given by

$$(\text{FEM})_{CB} = -0.0862(1.5)(40^2) = -206.88 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{BC} = 0.1133(1.5)(40^2) = 271.92 \text{ k} \cdot \text{ft}$$

Since member *AC* is prismatic

$$K_{CA} = \frac{4EI}{L_{AC}} = \frac{4E\left[\frac{1}{12}(1)(2)^3\right]}{40} = 0.0667E$$

Tabulating these data;

Joint	A	C		B
Mem	AC	CA	CB	BC
K		0.0667E	0.117E	
DF	0	0.3630	0.6370	0
COF	0	0.5	0.735	0
FEM			–206.88	
Dist.		75.10	131.78	271.92
CO	37.546			96.86
$\sum M$	37.546	75.10	–75.10	368.78

Thus,

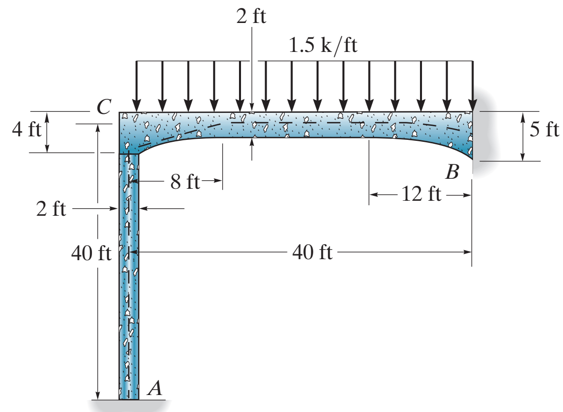
$M_{AC} = 37.55 \text{ k} \cdot \text{ft} = 37.6 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CA} = 75.10 \text{ k} \cdot \text{ft} = 75.1 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = -75.10 \text{ k} \cdot \text{ft} = -75.1 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BC} = 368.78 \text{ k} \cdot \text{ft} = 369 \text{ k} \cdot \text{ft}$ **Ans.**

***13–4.** Solve Prob. 13–3 using the slope-deflection equations.



The necessary data for member BC can be found from Table 13.2.

Here,

$$a_C = \frac{8}{40} = 0.2 \qquad a_B = \frac{12}{40} = 0.3 \qquad r_C = \frac{4-2}{2} = 1.0 \qquad r_B = \frac{5-2}{2} = 1.5$$

Thus,

$$C_{CB} = 0.735 \qquad C_{BC} = 0.589 \qquad K_{CB} = 7.02 \qquad K_{BC} = 8.76$$

Then,

$$K_{CB} = \frac{K_{CB}EI_C}{L_{BC}} = \frac{7.02E\left[\frac{1}{12}(1)(2)^3\right]}{40} = 0.117E$$

$$K_{BC} = \frac{K_{BC}EI_C}{L_{BC}} = \frac{8.76E\left[\frac{1}{12}(1)(2)^3\right]}{40} = 0.146E$$

The fixed end moment are given by

$$(\text{FEM})_{CB} = -0.0862(1.5)(40)^2 = -206.88 \text{ k}\cdot\text{ft}$$

$$(\text{FEM})_{BC} = 0.1133(1.5)(40)^2 = 271.92 \text{ k}\cdot\text{ft}$$

For member BC, applying Eq. 13–8,

$$M_N = K_N[\theta_N + C_N\theta_F - \psi(HC_N)] + (\text{FEM})_N$$

$$M_{CB} = 0.117E[\theta_c + 0.735(0) - 0(1 + 0.735)] + (-206.88) = 0.117E\theta_c - 206.88 \qquad (1)$$

$$M_{BC} = 0.146E[0 + 0.589\theta_C - 0(1 + 0.589)] + 271.92 = 0.085994E\theta_C + 271.92 \qquad (2)$$

Since member AC is prismatic, Eq. 11–8 is applicable

$$M_N = 2EK(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{AC} = 2E\left[\frac{\frac{1}{12}(1)(2)^3}{40}\right][2(0) + \theta_C - 3(0)] + 0 = 0.03333E\theta_C \qquad (3)$$

$$M_{CA} = 2E\left[\frac{\frac{1}{12}(1)(2)^3}{40}\right][2\theta_C + 0 - 3(0)] + 0 = 0.06667E\theta_C \qquad (4)$$

Moment equilibrium of joint C gives

$$M_{CA} + M_{CB} = 0$$

$$0.06667E\theta_C + 0.117E\theta_C - 206.88 = 0$$

$$\theta_C = \frac{1126.39}{E}$$

13–4. Continued

Substitute this result into Eqs. (1) to (4),

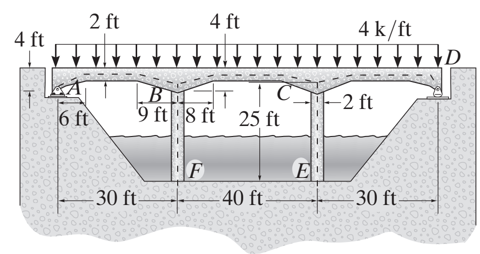
$M_{CB} = -75.09 \text{ k}\cdot\text{ft} = -75.1 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{BC} = 368.78 \text{ k}\cdot\text{ft} = 369 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{AC} = 37.546 \text{ k}\cdot\text{ft} = 37.5 \text{ k}\cdot\text{ft}$ **Ans.**

$M_{CA} = 75.09 \text{ k}\cdot\text{ft} = 75.1 \text{ k}\cdot\text{ft}$ **Ans.**

13–5. Use the moment-distribution method to determine the moment at each joint of the symmetric bridge frame. Supports at F and E are fixed and B and C are fixed connected. Use Table 13–2. Assume E is constant and the members are each 1 ft thick.



For span AB,

$$a_A = \frac{6}{30} = 0.2 \qquad a_B = \frac{9}{30} = 0.3$$

$$r_A = r_B = \frac{4-2}{2} = 1$$

From Table 13–2,

$C_{AB} = 0.683 \qquad C_{BA} = 0.598$

$k_{AB} = 6.73 \qquad k_{BA} = 7.68$

$$K_{AB} = \frac{6.73EI}{30} = 0.2243EI$$

$$K_{BA} = \frac{7.68EI}{30} = 0.256EI$$

$$K_{BA} = 0.256EI[1-(0.683)(0.598)] = 0.15144EI$$

$(\text{FEM})_{AB} = -0.0911(4)(30)^2 = -327.96 \text{ k}\cdot\text{ft}$

$(\text{FEM})_{BA} = 0.1042(4)(30)^2 = 375.12 \text{ k}\cdot\text{ft}$

13–5. Continued

For span CD,

$C_{DC} = 0.683 \qquad C_{CD} = 0.598$

$K_{DC} = 6.73 \qquad K_{CD} = 7.68$

$K_{DC} = 0.2243EI$

$K_{CD} = 0.256EI$

$K_{CD} = 0.15144EI$

$(\text{FEM})_{CD} = -375.12 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{DC} = 327.96 \text{ k} \cdot \text{ft}$

For span BC,

$$a_B = a_C = \frac{8}{40} = 0.2$$

$$r_A = r_{CB} = \frac{4 - 2}{2} = 1$$

From Table 13–2,

$C_{BC} = C_{CB} = 0.619$

$k_{BC} = k_{CB} = 6.41$

$$K_{BC} = K_{CB} = \frac{6.41EI}{40} = 0.16025EI$$

$(\text{FEM})_{BC} = -0.0956(4)(40)^2 = -611.84 \text{ k} \cdot \text{ft}$

$(\text{FEM})_{CB} = 611.84 \text{ k} \cdot \text{ft}$

For span BF,

$C_{BF} = 0.5$

$$K_{BF} = \frac{4EI}{25} = 0.16EI$$

$(\text{FEM})_{BF} = (\text{FEM})_{FB} = 0$

For span CE,

$C_{CE} = 0.5$

$K_{CE} = 0.16EI$

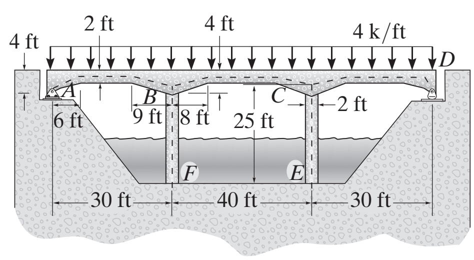
$(\text{FEM})_{CE} = (\text{FEM})_{EC} = 0$

13–5. Continued

Joint	*A*	*F*	*B*			*C*			*E*	*D*
Member	*AB*	*FB*	*BF*	*BA*	*BC*	*CB*	*CD*	*CE*	*EC*	*DC*
DF	1	0	0.3392	0.3211	0.3397	0.3397	0.3211	0.3392	0	1
COF	0.683		0.5	0.598	0.619	0.619	0.598	0.5		0.683
FEM	−327.96			375.12	−611.84	611.84	−375.12			332.96
	327.96		80.30	76.01	80.41	−80.41	−76.01	−80.30		−327.96
		40.15		224.00	−49.77	49.77	−224.00		−40.15	
			−59.09	−55.95	−59.19	59.19	55.95	59.19		
		−29.55			36.64	−36.64			29.55	
			−12.42	−11.77	−12.45	12.45	11.77	12.42		
		−6.21			7.71	−7.71			6.21	
			−2.61	−2.48	−2.62	2.62	2.48	2.61		
		−1.31			1.62	−1.62			1.31	
			−0.55	−0.52	−0.55	0.55	0.52	0.55		
		−0.27			0.34	−0.34			−0.27	
			−0.11	−0.11	−0.12	0.12	0.11	0.11		
		−0.5			0.07	−0.07			0.05	
			−0.03	−0.02	−0.02	0.02	0.02	0.03		
$\sum$	0	2.76	5.49	604	−609	609	−604	5.49	−2.76	0

k · ft **Ans.**

13–6. Solve Prob. 13–5 using the slope-deflection equations.



See Prob. 13–19 for the tabulated data

$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 - C_N)] + (\text{FEM})_N$

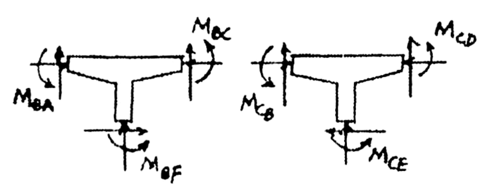
For span *AB*,

$M_{AB} = 0.2243EI(\theta_A + 0.683\theta_B - 0) - 327.96$

$M_{AB} = 0.2243EI\theta_A + 0.15320EI\theta_B - 327.96$ (1)

$M_{BA} = 0.256EI(\theta_B + 0.598\theta_A - 0) + 375.12$

$M_{BA} = 0.256EI\theta_B + 0.15309EI\theta_A + 375.12$ (2)



For span *BC*,

$M_{BC} = 0.16025EI(\theta_B + 0.619\theta_C - 0) - 611.84$

$M_{BC} = 0.16025EI\theta_B + 0.099194EI\theta_C - 611.84$ (3)

$M_{CB} = 0.16025EI(\theta_C + 0.619\theta_B - 0) + 611.84$

$M_{CB} = 0.16025EI\theta_C + 0.099194EI\theta_B + 611.84$ (4)

13–6. Continued

For span *CD*,

$M_{CD} = 0.256EI(\theta_C + 0.598\theta_D - 0) - 375.12$

$M_{CD} = 0.256EI\theta_C + 0.15309EI\theta_D - 375.12$ (5)

$M_{DC} = 0.2243EI(\theta_D + 0.683\theta_C - 0) + 327.96$

$M_{DC} = 0.2243EI\theta_D + 0.15320EI\theta_C + 327.96$ (6)

For span *BF*,

$M_{BF} = 2E\left(\frac{1}{25}\right)(2\theta_B + 0 - 0) + 0$

$M_{BF} = 0.16EI\theta_B$ (7)

$M_{FB} = 2E\left(\frac{1}{25}\right)(2(0) + \theta_B - 0) + 0$

$M_{FB} = 0.08EI\theta_B$ (8)

For span *CE*,

$M_{CE} = 2E\left(\frac{1}{25}\right)(2\theta_C + 0 - 0) + 0$

$M_{CE} = 0.16EI\theta_C$ (9)

$M_{EC} = 2E\left(\frac{1}{25}\right)(2(0) + \theta_C - 0) + 0$

$M_{EC} = 0.08EI\theta_C$ (10)

Equilibrium equations:

$M_{AB} = 0$ (11)

$M_{DC} = 0$ (12)

$M_{BA} + M_{BC} + M_{BF} = 0$ (13)

$M_{CB} + M_{CE} + M_{CD} = 0$ (14)

Solving Eq. 1–14,

$\theta_A = \frac{1438.53}{EI} \qquad \theta_B = \frac{34.58}{EI} \qquad \theta_C = \frac{-34.58}{EI} \qquad \theta_D = \frac{-1438.53}{EI}$

$M_{AB} = 0$ **Ans.**

$M_{BA} = 604\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BC} = -610\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BF} = 5.53\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{FB} = 2.77\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CB} = 610\ \text{k}\cdot\text{ft}$ **Ans.**

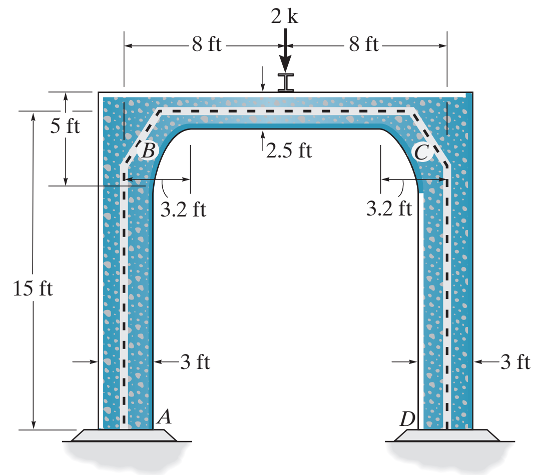
$M_{CD} = -604\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CE} = -5.53\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{EC} = -2.77\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{DC} = 0$ **Ans.**

13–7. Apply the moment-distribution method to determine the moment at each joint of the symmetric parabolic haunched frame. Supports A and D are fixed. Use Table 13–2. The members are each 1 ft thick. E is constant.



$$a_B = a_C = \frac{3.2}{16} = 0.2$$

$$r_B = r_C = \frac{5 - 2.5}{2.5} = 1$$

$$C_{BC} = C_{CB} = 0.619$$

$$k_{BC} = k_{CB} = 6.41$$

$$(\text{FEM})_{BC} = -0.1459(2)(16) = -4.6688 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = 4.6688 \text{ k} \cdot \text{ft}$$

$$K_{BC} = K_{CB} = \frac{k_{BC}EI_C}{L} = \frac{6.41(E)\left(\frac{1}{12}\right)(1)(2.5)^3}{16} = 0.5216E$$

$$K_{BA} = K_{CO} = \frac{4EI}{L} = \frac{4E\left[\frac{1}{12}(1)(3)^3\right]}{15} = 0.6E$$

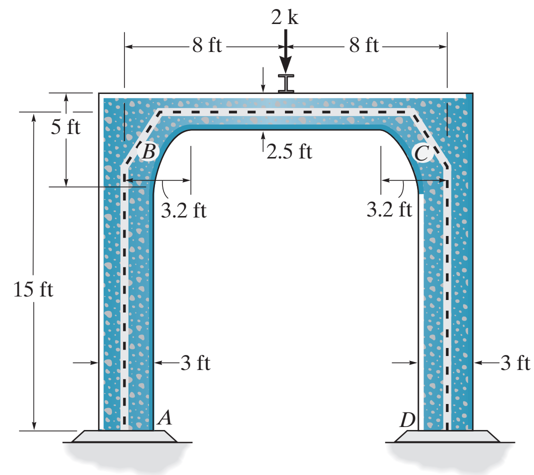
$$(\text{DF})_{BA} = (\text{DF})_{CD} = \frac{0.6E}{0.5216E + 0.6E} = 0.535$$

$$(\text{DF})_{BC} = (\text{DF})_{CB} = 0.465$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	0	0.535	0.465	0.465	0.535	0
COF	0.5	0.5	0.619	0.619	0.5	0.5
FEM			−4.6688	4.6688		
		2.498	2.171	−2.171	−2498	
	1.249		−1.344	1.344		−1.249
		0.7191	0.6249	−0.6249	−0.7191	
	0.359		−0.387	0.387		−0.359
		0.207	0.180	−0.180	−0.207	
	0.103		−0.111	0.111		−0.103
		0.059	0.052	−0.052	−0.059	
	0.029		−0.032	0.032		−0.029
		0.017	0.015	−0.015	−0.017	
	0.008		−0.009	0.009		−0.008
		0.005	0.004	−0.004	−0.005	
	0.002		−0.002	0.002		0.002
		0.001	0.001	−0.001	−0.001	
$\sum$	1.750	3.51	−3.51	3.51	−3.51	−1.75 k · ft

Ans.

***13–8.** Solve Prob. 13–7 using the slope-deflection equations.



$$a_B = a_C = \frac{3.2}{16} = 0.2$$

$$r_B = r_C = \frac{5 - 2.5}{2.5} = 1$$

$$C_{BC} = C_{CB} = 0.619$$

$$k_{BC} = k_{CB} = 6.41$$

$$(\text{FEM})_{BC} = 0.1459(2)(16) = -4.6688 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{CB} = -4.6688 \text{ k} \cdot \text{ft}$$

$$K_{BC} = K_{CB} = \frac{k_{BC} E I_c}{L} = \frac{6.41(E)\left(\frac{1}{12}\right)(1)(2.5)^2}{16} = 0.5216E$$

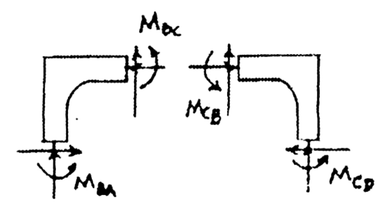
$$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 + C_N)] + (\text{FEM})_N$$

$$M_{AB} = \frac{2EI}{15}(0 + \theta_B - 0) + 0$$

$$M_{BA} = \frac{2EI}{15}(2\theta_B + 0 - 0) + 0$$

$$M_{CD} = \frac{2EI}{15}(2\theta_C + 0 - 0) + 0$$

$$M_{DC} = \frac{2EI}{15}(0 + \theta_C - 0) + 0$$



$$M_{BC} = 0.5216E(\theta_B + 0.619(\theta_C) - 0) - 4.6688$$

$$M_{CB} = 0.5216E(\theta_C + 0.619(\theta_B) - 0) + 4.6688$$

Equilibrium.

$$M_{BA} + M_{BC} = 0$$

$$M_{CB} + M_{CD} = 0$$

Or,

$$\frac{2E\left(\frac{1}{12}\right)(1)(3)^3}{15}(2\theta_B) + 0.5216E[\theta_B + 0.619\theta_C] - 4.6688 = 0$$

$$1.1216\theta_B + 0.32287\theta_C = \frac{4.6688}{E} \qquad (1)$$

$$\frac{2E\left(\frac{1}{12}\right)(1)(3)^3}{15}(2\theta_C) + 0.5216E[\theta_C + 0.619\theta_B] + 4.6688 = 0$$

$$1.1216\theta_C + 0.32287\theta_B = -\frac{4.6688}{E} \qquad (2)$$

13–8. Continued

Solving Eqs. 1 and 2:

$$\theta_B = -\theta_C = \frac{5.84528}{E}$$

$M_{AB} = 1.75 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{BA} = 3.51 \text{ k} \cdot \text{ft}$ **Ans.**

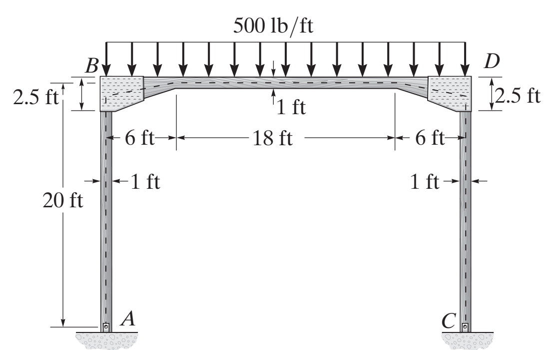
$M_{BC} = -3.51 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = 3.51 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CD} = -3.51 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{DC} = -1.75 \text{ k} \cdot \text{ft}$ **Ans.**

13–9. Use the moment-distribution method to determine the moment at each joint of the frame. The supports at A and C are pinned and the joints at B and D are fixed connected. Assume that E is constant and the members have a thickness of 1 ft. The haunches are straight so use Table 13–1.



For span BD,

$$a_B = a_D = \frac{6}{30} = 0.2$$

$$r_A = r_B = \frac{2.5 - 1}{1} = 1.5$$

From Table 13–1,

$$C_{BD} = C_{DB} = 0.691$$

$$k_{BD} = k_{DB} = 9.08$$

$$K_{BD} = K_{DB} = \frac{kEI_C}{L} = \frac{9.08EI}{30} = 0.30267EI$$

$$(\text{FEM})_{BD} = -0.1021(0.5)(30^2) = -45.945 \text{ k} \cdot \text{ft}$$

$$(\text{FEM})_{DB} = 45.945 \text{ k} \cdot \text{ft}$$

For span AB and CD,

$$K_{BA} = K_{DC} = \frac{3EI}{20} = 0.15EI$$

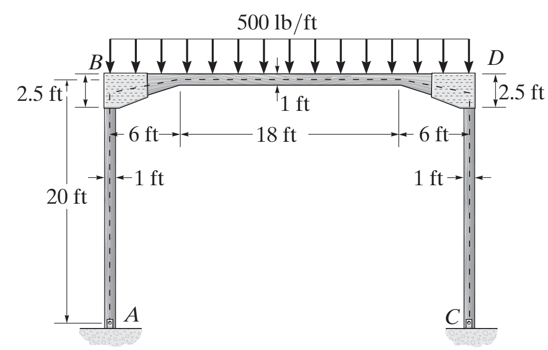
$$(\text{FEM})_{AB} = (\text{FEM})_{BA} = (\text{FEM})_{DC} = (\text{FEM})_{CD} = 0$$

13–9. Continued

Joint	A	B		D		C
Mem.	AB	BA	BD	DB	DC	CD
K		$0.15EI$	$0.3026EI$	$0.3026EI$	$0.15EI$	
DF	1	0.3314	0.6686	0.6686	0.3314	1
COF		0	0.691	0.691	0	
FEM			−45.95	45.95		
		15.23	30.72	−30.72	−15.23	
			−21.22	21.22		
		7.03	14.19	−14.19	−7.03	
			−9.81	9.81		
		3.25	6.56	−6.56	−3.25	
			−4.53	4.53		
		1.50	3.03	−3.03	−1.50	
			−2.09	2.09		
		0.69	1.40	−1.40	−0.69	
			−0.97	0.97		
		0.32	0.65	−0.65	−0.32	
			−0.45	0.45		
		0.15	0.30	−0.30	−0.15	
			−0.21	0.21		
		0.07	0.14	−0.14	−0.07	
			−0.10	0.10		
		0.03	0.06	−0.06	−0.03	
			−0.04	0.04		
		0.01	0.03	−0.03	−0.01	
$\sum M$	0	28.3	−28.3	28.3	−28.3	0 k · ft

Ans.

13–10. Solve Prob. 13–9 using the slope-deflection equations.



See Prob. 13–17 for the tabular data.

For span AB,

$$M_N = 3E\frac{I}{L}[\theta_N - \psi] + (\text{FEM})_N$$

$$M_{BA} = 3E\left(\frac{I}{20}\right)(\theta_B - 0) + 0$$

$$M_{BA} = \frac{3EI}{20}\theta_B \qquad (1)$$

13–10. Continued

For span BD,

$M_N = K_N[\theta_N + C_N\theta_F - \psi(1 + C_N)] + (\text{FEM})_N$

$M_{BD} = 0.30267EI(\theta_B + 0.691\theta_D - 0) - 45.945$

$M_{BD} = 0.30267EI\theta_B + 0.20914EI\theta_D - 45.945$ (2)

$M_{DB} = 0.30267EI(\theta_D + 0.691\theta_B - 0) + 45.945$

$M_{DB} = 0.30267EI\theta_D + 0.20914EI\theta_B - 45.945$ (3)

For span DC,

$M_N = 3E\dfrac{I}{L}[\theta_N - \psi] + (\text{FEM})_N$

$M_{DC} = 3E\left(\dfrac{I}{20}\right)(\theta_D - 0) + 0$

$M_{DC} = \dfrac{3EI}{20}\,\theta_D$ (4)

Equilibrium equations,

$M_{BA} + M_{BD} = 0$ (5)

$M_{DB} + M_{DC} = 0$ (6)

Solving Eqs. 1–6:

$\theta_B = \dfrac{188.67}{EI} \qquad \theta_D = -\dfrac{188.67}{EI}$

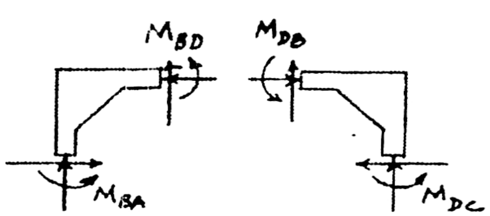
$M_{BA} = 28.3\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{BD} = -28.3\ \text{k}\cdot\text{ft}$ **Ans.**

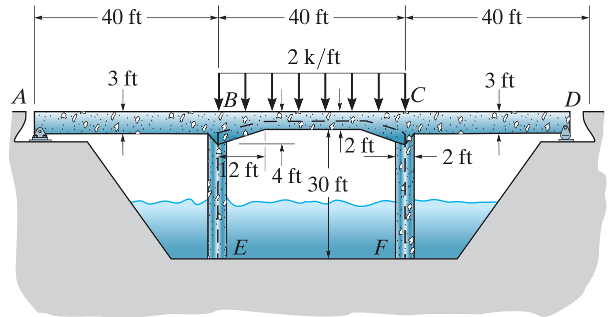
$M_{DB} = 28.3\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{DC} = -28.3\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{AB} = M_{CD} = 0$ **Ans.**



13–11. Use the moment-distribution method to determine the moment at each joint of the symmetric bridge frame. Supports F and E are fixed and B and C are fixed connected. The haunches are straight so use Table 13–2. Assume E is constant and the members are each 1 ft thick.



The necessary data for member BC can be found from Table 13–1.

Here,

$$a_B = a_C = \frac{12}{40} = 0.3$$

$$r_B = r_C = \frac{4-2}{2} = 1.0$$

Thus,

$$C_{BC} = C_{CB} = 0.705 \qquad K_{BC} = K_{CB} = 10.85$$

Since the stimulate and loading are symmetry, Eq. 13–14 applicable.

Here,

$$K_{BC}\frac{K_{BC}EI_C}{L_{BC}} = \frac{10.85E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.18083E$$

$$K'_{BC} = K_{BC}(1 - C_{BC}) = 0.18083E(1 - 0.705) = 0.05335E$$

The fixed end moment are given by

$$(\text{FEM})_{BC} = -0.1034(2)(40^2) = -330.88 \text{ k} \cdot \text{ft}$$

Since member AB and BE are prismatic

$$K_{BE} = \frac{4EI}{L_{BA}} = \frac{4E\left[\frac{1}{12}(1)(2^3)\right]}{30} = 0.08889E$$

$$K_{BA} = \frac{3EI}{L_{BA}} = \frac{3E\left[\frac{1}{12}(1)(3^3)\right]}{40} = 0.16875E$$

Tabulating these data,

Joint	A	B			E
Member	AB	BA	BC	BE	EB
K		$0.16875E$	$0.05335E$	$0.08889E$	
DF	1	0.5426	0.1715	0.2859	0
COF		0	0.705	0.5	
FEM Dist		 179.53	−330.88 56.75	 94.60	
CO					47.30
$\sum M$		179.53	−274.13	94.60	47.30

13–11. Continued

Thus,

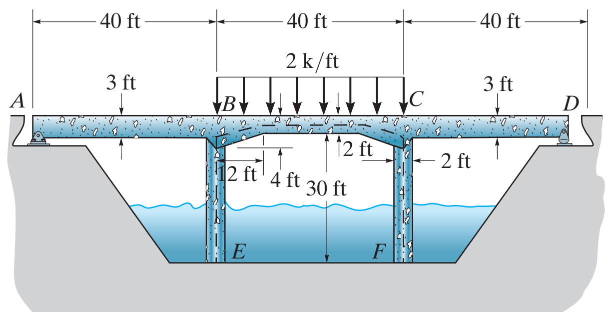
$M_{CD} = M_{BA} = 179.53 \text{ k} \cdot \text{ft} = 180 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CF} = M_{BE} = 94.60 \text{ k} \cdot \text{ft} = 94.6 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{CB} = M_{BC} = -274.13 \text{ k} \cdot \text{ft} = 274 \text{ k} \cdot \text{ft}$ **Ans.**

$M_{FC} = M_{EB} = 47.30 \text{ k} \cdot \text{ft} = 47.3 \text{ k} \cdot \text{ft}$ **Ans.**

***13–12.** Solve Prob. 13–11 using the slope-deflection equations.



The necessary data for member BC can be found from Table 13–1

Here,

$$a_B = a_C = \frac{12}{40} = 0.3 \qquad r_B = r_C = \frac{4-2}{2} = 1.0$$

Thus,

$$C_{BC} = C_{CB} = 0.705 \qquad K_{BC} = K_{CB} = 10.85$$

Then,

$$K_{BC} = K_{CB} = \frac{K_{BC}EI_C}{L_{BC}} = \frac{10.85E\left[\frac{1}{12}(1)(2^3)\right]}{40} = 0.1808E$$

The fixed end moment's are given by

$$(\text{FEM})_{BC} = -0.1034(2)(40^2) = -330.88 \text{ k} \cdot \text{ft}.$$

For member BC, applying Eq. 13–8. Here, due to symatry,

$\theta_C = -\theta_B$

$$M_N = K_N[\theta_N + C_N\theta_F - \psi(HC_N)] + (\text{FEM})_N$$

$$M_{BC} = 0.1808E[\theta_B + 0.705(-\theta_B) - 0\,(1 + 0.705)] + (-330.88)$$
$$= 0.053346E\theta_B - 330.88 \qquad (1)$$

13–12. Continued

For prismatic member BE, applying Eq. 11–8.

$$M_N = 2EK(2\theta_N + \theta_F - 3\psi) + (\text{FEM})_N$$

$$M_{BE} = 2E\left[\frac{\frac{1}{12}(1)(3)^3}{30}\right][2\theta_B + 0 - 3(0)] + 0 = 0.08889E\theta_B \qquad (2)$$

$$M_{EB} = 2E\left[\frac{\frac{1}{12}(1)(2)^3}{30}\right][2(0) + \theta_B - 3(0) + 0 = 0.04444E\theta_B \qquad (3)$$

For prismatic member AB, applying Eq. 11–10

$$M_N = 3EK(\theta_N - \psi) + (\text{FEM})_N$$

$$M_{BA} = 3E\left[\frac{\frac{1}{12}(1)(2)^3}{40}\right](\theta_B - 0) + 0 = 0.16875E\theta_B \qquad (4)$$

Moment equilibrium of joint B gives

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$0.16875E\theta_B + 0.053346E\theta_B - 330.88 + 0.08889E\theta_B = 0$$

$$\theta_B = \frac{1063.97}{E}$$

Substitute this result into Eq. (1) to (4)

$M_{CB} = M_{BC} = -274.12\ \text{k}\cdot\text{ft} = -274\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CF} = M_{BE} = 94.58\ \text{k}\cdot\text{ft} = 94.6\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{FC} = M_{EB} = 47.28\ \text{k}\cdot\text{ft} = 47.3\ \text{k}\cdot\text{ft}$ **Ans.**

$M_{CD} = M_{BA} = 179.55\ \text{k}\cdot\text{ft} = 180\ \text{k}\cdot\text{ft}$ **Ans.**